

How Petrobras lost more than BRL 181 billion? A valuation analysis in 2010-2016.

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Resumo

Este artigo analisa uma série com os resultados de um modelo de avaliação das ações da Petrobras em 2010-2016. Nosso principal objetivo é responder à seguinte pergunta: quanto foi a perda de valor da Petrobras nesse período marcada pela mudança crucial do regime de concessão para o regime de compartilhamento? Utilizamos três critérios para avaliar essa perda: (i) estimativa de uma série de valores intrínsecos do PETR3 usando o modelo de avaliação Damodaran; (ii) comparação com a série de valores de mercado da companhia; (iii) cálculo de uma série de valores corporativos da empresa. A comparação usando três critérios visa oferecer ao leitor três métricas que, quando comparadas, permitem que ele tenha uma ideia clara de quanto a empresa foi afetada pelos graves erros de gerenciamento que destruíram uma parte expressiva de seu valor.

Palavras-chave: valuation de ações, fluxo de caixa descontado, modelo de valuation de Damodaran, Petrobras.

Abstract

This paper analyzes a series with the results of an evaluation model for Petrobras' actions in 2010-2016. Our main objective is to answer the following question: how much was Petrobras' loss of value in this period marked by the crucial change from the concession regime to the sharing regime? We used three criteria to assess this loss: (i) estimation of a series of intrinsic values of PETR3 using the Damodaran valuation model; (ii) comparison with the company's market values series; (iii) calculation of a series of the company's enterprise values. The comparison by using three criteria aims to offer to the reader with three metrics that, when compared, allow him to give a clear idea of how much the company was affected by the serious management errors that destroyed an expressive part of its value.

Keywords: stock valuation, discounted cash flow, Damodaran valuation model, Petrobras.

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"Financial assets are acquired by their expected cash flows." Aswath Damodaran

1. Introduction

In the second section of this paper, after the introduction, we present a theoretical and empirical review of literature on valuation techniques and models.

In the third section, we describe the Damodaran's valuation model (1994).

In the fourth section, we analyze the model results for the mentioned period and answer two questions: (i) what factors has conditioned the upside and downside cycles of PETR3 prices and their intrinsic values? (ii) how much was the Petrobras' lost value in this period marked by the crucial change from the concession regime to the sharing regime?

In the fifth and final section, we make the final considerations.

2. Review of literature

The first subsection reviews the theoretical literature and the second, the empirical.

2.1. Theoretical Literature

We examine in this subsection the theory and evidence of different approaches to valuation (hereafter, simply "valuation models"). The focus is on the discounted cash flow valuation models.

Bodie, Kane and Marcus (2009, 589) consider that the intrinsic value of a stock (the goal of valuation techniques) corresponds to the buyer's present value of expected cash flow. This projected cash flow must include dividends and amounts arising from the final sale of the shares, discounted at an appropriate risk-adjusted interest rate.

With a similar but broader view, Damodaran (2006, p. 3) considers asset valuation to be at the heart of many analytical activities in Finance. It is useful both in the study of market efficiency as in the analysis of corporate governance issues, as well in the comparison of different investment decision rules for capital budget.

Analysts usually use a set of valuation models. There are those that are simple and others are sophisticated. Some examples: i) discounted cash flow model - the focus of this paper - relates the value of an asset to their present value of expected future cash flows; ii) settlement and accounting evaluation model, which evaluate the existing assets of a company, using accounting estimates of value or the own book value as a "start point" of the process; (iii) a model that estimates the value of an asset taking into account the pricing of other "comparable" assets in relation to a common variable (such as profits, cash flows, book value or sales); iv) a contingent valuation model that measure the value of the assets that resemble to, in general or in part, the options (this class of models is related to the real options theory).

According to Parker (1968), the pioneering interest rate tables date back to 1340. He also attributes to the first publication on the subject, the "Pratica della Mercatura" of 1766, to Francesco Pegolotti, a Florentine businessman and politician.

However, the seminal contributions to discounted cash flow valuation techniques were established by Alfred Marshall (1907) and Bohm-Bawerk (1903). Both explored the notion of present value in their works in the early twentieth century. They influenced Irving Fisher, who developed this conception and sophisticated it in *The Rate of Interest* of 1907 and *The Theory of Interest* of 1930. In both works, Fisher proposed four alternative approaches to analyzing investments. According to him, they would generate the same results. He argued that, when faced with various investment alternatives, one should choose the investment: (i) which has the highest present value at the market interest rate; ii) whose present value of the benefits exceeds the present value of the costs; (iii) whose "rate of return on sacrifice" exceeds the market interest rate; (iv) that compared to the similar most expensive investment, generate a rate of return on the cost higher than the market interest rate. It should be noted that the first two approaches represent the net present value rule. The third is a variant of the internal rate of return (IRR) approach. And the latter corresponds to the marginal rate of return approach.

As Fisher did not deeply explore the notion of the rate of return, other economists better explored this idea. Starting from the analysis of a single investment, Boulding (1935) deduced the internal rate of return of an investment from its expected cash flows and from an initial investment.

Keynes (1936) argued that the "marginal efficiency of capital" could be calculated as the discount rate that equal the present value of an asset's returns to its current price. It

is equivalent to internal rate of return of an investment (the same rate developed by Fisher, in practical terms).

Samuelson (1937) explored the differences between the internal rate of return (IRR) and net present value approaches. He also argued that rational investors should maximize the net present value, not the IRR.

In the past 50 years, discounted cash flow models become popular among financiers and businessmen and have expanded their scope for insurance and business valuation. According to Damodaran (2006), this impulse was stimulated by the developments of portfolio theory, on the one hand. On the other hand, the rise of billionaires who use the fundamentalist analysis, among them the famous Warren Buffett, contributed to the success of biographical best sellers like Hagstrom (2004). Because Buffett's investment philosophy is inspired by Benjamin Graham and Phillip Fisher, predecessor investors, the works of Dodd and Graham (1934 and 1949) and Fisher (1960) received many reissues and helped to spark interest on the methods of discounted cash flows. For fundamentalists (and unlike chartists), the intrinsic value of a stock differs of its price. So, investors seek to know, by discounted cash flow models and other techniques, how the value oscillations occur. Based on these analysis, they continually try to anticipate possible price changes. The classic approach is the dividend discount model, which is the basis of corporate finance theory. This model postulates that the company value is the sum of all expected dividend payments minus their present net value. However, these components are uncertain.

Another theory is that the company value is defined by an "efficient market". According to Malkiel (2012), the actions follow a "random walk" in which they incorporate all the information available in the stock prices. In this sense, there is no way to get a consistent result better than the market itself.

In addition to the dividend discount model, fundamentalist analysts use other instruments such: (a) the price-to-earnings ratio (P/E); (b) the profitability of the dividends; (c) the price-to-book ratio that compares the market value with the book value; And (d) the "Tobin's q" which is the relation between market value and the replacement cost of the assets. But there are other measures. There are also other criteria for comparing stock values with other financial assets values, such as: the "Fed model", which divides the stock profitability by the profitability of US Treasury bonds (if the result is smaller than one, the shares can be considered with attractive price for the purchase); The "stock risk premium", which measures the extra return of shares on the return of government

bonds. In addition, there are other volatility indicators that are applicable to individual stocks, Such as beta index, which measures the volatility of a share in relation to the market as a whole. Smaller companies' stocks tend to have higher betas, since they have higher risk and lower liquidity than the larger or blue chips ones. Because they have higher betas, they can represent more attractive returns than those of larger companies that usually have smaller betas.

However, none of these indicators is free of weaknesses. Indicators that use profits are affected by "little tricks" in their accounting. Those that measure profitability are affected by periods of low inflation, when the supply of government bonds decrease and government bonds are also not absolutely risk free, being not uncommon to find investors affected by declines in their real returns in periods of high inflation.

"Tobin's q" does not take into account intangible capital. The intangible components of the asset can be very valuable given the assessments of brands and patents. The stock risk premium was considered by economists to be somewhat inaccurate, since it has long remained above the long-term stock return. In the twentieth century the US stocks long-term return was near to 6% (Siegel, 2013). Furthermore, it can be measured only "a posteriori".

The "beta", a widely used index, was also questioned, since the return of small company stocks did not stay above the stock returns of the majors during the periods of high registered in the decades of 80 and 90 of the last century. In sum, it is always possible to find arguments against the use of these indicators.

But still, all of these pricing metrics helped identify bull market US stock markets during the nearly three decades (80, 90 and 2000-2007), which have reached very high historical levels. They were also useful for analyzing the most recent periods of expressive high in Ibovespa (2004-2005 and 2006-2008).

2.2. Empirical literature: most researched valuation models

There are three widely used methods of valuation of companies by discounted cash flow, according to Fernández (2008, pp. 192-196). They always generate the same value, since they analyze the same situation under the same hypotheses, but using different flows for the valuation process.

2.2.1. The first method

The first method is the stock's flow discounted of required profitability of it.

Expression (1) indicates that the value of the stocks (E) corresponds to the net present value of expected stocks' flows discounted by the required profitability of the shares.

$$(1) E_0 = NPV_0(CFac_t; Ke_t)$$

By (2) we have that the debt value (D) is equivalent to the net present value of debt expected flows discounted of the required profitability for the debt. In expression (3), the CFd is described as the expected value (at t = 0) of debt flow in t.

$$(2) D_0 = NPV_0(CFd_t; Kd_t)$$

$$(3) CFd_t = D_{t-1}Kd_t - (D_t - D_{t-1})$$

2.2.2. *The second method*

The second method is the free cash flow discounted by WACC (Weighted Average Cost of Capital).

Formula (4) indicates that the value of debt (D) plus stock value (E) is equal to the current value of the company's expected free cash flows, discounted from the weighted cost of debt and own capital deducted of taxes (WACC).

$$(4) E_0 + D_0 = NPV_0(FCF_t; WACC_t)$$

The expression that combine FCF and CFac is :

$$(5) CFac_t = FCF_t + \Delta D_t - D_{t-1} \cdot Kd_t (1 - T_t)$$

Where, ΔD_t represents a change in debt value; and $D_{t-1} \cdot Kd_t$ are the interest paid by the company in t.

The definition of WACC says that it is the rate at which the FCF should be discounted so that equation (4) gives the same result as the sum of (1) and (2) that was effected in the first method.

That is, the intertemporal expression of equations (1), (2) and (4) is:

$$(1') E_t = E_{t-1} (1 + Ke_t) - CFac_t$$

$$(2'') D_t = D_{t-1} (1 + Kd_t) - CFd_t$$

$$(4''') E_t + D_t = (E_{t-1} + D_{t-1}) (1 + WACC_t) - FCF_t$$

Subtracting (4i) from the sum of (1i) and (2i) gives:

$$0 = E_{t-1}Ke_t + D_{t-1}Kd_t - (E_{t-1} + D_{t-1})WACC_t + FCF_t - CFac_t - CFd_t$$

From (4''') we can deduce that $FCF_t - CFac_t - CFd_t = -D_{t-1}Kd_tT_t$, therefore, we get (6):

$$WACC_t = \frac{E_{t-1}Ke_t + D_{t-1}Kd_t(1 - T_t)}{E_{t-1} + D_{t-1}} \quad (6)$$

Where: E_{t-1} is the stock's value at previous period; Ke_t is the required stock profitability; Kd_t correspond to debt cost and T_t is the effective tax burden on the company.

Equation (6) has a special meaning. So, let's deduce it again considering a perpetuity that grows over time under a rate g . Under this hypothesis, we have (4) modified to:

$$E_0 + D_0 = \frac{FCF_1}{(WACC - g)}$$

In a perpetuity we have $CFac_1 = E_0(Ke - g)$, therefore:

$$(E_0 + D_0)(WACC - g) = [E_0Ke + D_0Kd(1 - T)] - g(E_0 + D_0)$$

And after simplifying:

$$WACC = \frac{[E_0Ke + D_0Kd(1 - T)]}{(E_0 + D_0)}$$

2.2.3. The third method

The third approach is the adjusted present value (APV).

In (7), considering a levered company, debt value (D) plus its stocks' value (E) is identical to the stocks' value of a unlevered company plus the net present value of its tax savings resulting from reduction in taxable income achieved through claiming allowable

deductions such as mortgage interest, medical expenses, charitable donations, amortization and depreciation (value of tax shields - VTS).

$$(7) \quad E_0 + D_0 = Vu_0 + VTS$$

Being the K_u the required profitability of an unlevered company's stocks, V_u , or the unlevered company share's value, is given by the net present value of FCF_t (the free cash flow in t) divided by K_u :

$$(8) \quad Vu = NPV_0(FCF_t, Ku_t)$$

So, it is clear that relation between K_u , K_e and WACC depend on VTS.

And the identities that relate betas to expected returns are:

$$(9) \quad Ke = R_F + \beta_L P_M ; Kd = R_F + \beta_d P_M \text{ and } Ku = R_F + \beta_u P_M .$$

$$(10)$$

Where: R_F is the risk-free rate and P_M is the prime risk market rate.

Finally, the most commonly used valuation models are Myers (1974), Miles and Ezzell (1980), Harris, Pringle and Ruback (1985 and 1995), Damodaran (1994) and Fernández (2004 and 2008).

They adopt similar equations with slight modifications, but the presentation of these models is not the focus of this work.

However, in order to demonstrate this similarity between the most researched and used valuation models, we summarize in Table 1 the equations that structure them and in the next section, we describe the Damodaran (1994) model.

Table 1- Equations used in the most used valuation models

Equations	Meanings
$Ku = R_F + \beta_u P_M$	K_u is the required profitability of the shares of the company not levered (with $D = 0$). R_F is the risk-free rate. β_u is the unlevered beta.
1) $E_0 = \sum_{t=1}^{\infty} \frac{CFac_t}{(1 + Ke_1) \dots (1 + Ke_t)}$	E_0 is the stock's value in $t = 0$. $CFac_t$ is the shareholders' expected cash flow in t. Ke is required profitability of the

$$2) \quad D_0 = \sum_{t=1}^{\infty} \frac{CFd_t}{(1+Kd_1)\dots(1+Kd_t)}$$

$$3) \quad (1) \quad + \quad (2) \quad \text{or} \\
E_0 + D_0 = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC_1)\dots(1+WACC_t)}$$

$$4) \quad WACC_t = \frac{E_{t-1} \cdot Ke_t + D_{t-1} \cdot Kd_t \cdot (1+T_t)}{E_{t-1} + D_{t-1}}$$

$$5) \quad (3) - D_0 = E_0$$

$$6) \quad VTS = TKu \sum_{t=1}^{\infty} \frac{D_{t-1}}{(1+Ku)^t}$$

$$7) \quad Vu_0 = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+Ku_1)\dots(1+Ku_t)}$$

$$8) \quad (6)+(7) = VTS + Vu_0$$

$$9) \quad (8) - D_0 = E_0$$

Source: Fernández (2008, p. 204).

3. Describing Damodaran's valuation model (1994)

The description of the model begins with VTS specification:

$$VTS = NPV [Ku; DKu - D(Kd - R_f)(1-T)] \quad (1)$$

¹ Such deductions reduce part of the taxpayer's taxable income in a given year or transfer tax debts to future years (Fernández, 2004 and 2006). So, the tax benefit reduces the total amount of the company's tax liability.

company's stocks along periods 1 to t (or the cost of own resources).

D_0 is the debt in the current period; CFd is the debt's expected cash flow; Kd is the required profitability of the company's debt (being $Kd = R_f + \beta_d P_M$, where β_d is the beta applied on company's debt).

FCF_t is the free cash flow (equivalent to $CFac$, if, for instance, the company have no debts).

$WACC$ is the weighted average cost of capital. T_t is the tax burden on the company in t. The other variables have already been mentioned.

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VTS is the value of tax shields, i.e., the reduction of the taxable income of an individual or legal entity. This reduction is obtained through legal deductions of interest payments, medical expenses, philanthropic donations, depreciation, amortization, etc.¹

Vu is the current value of the unlevered company's shares (a hypothetical firm with $D = 0$).

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Where VTS is the value of tax shields, i.e., the reduction of the taxable income of an individual or legal entity. This reduction is obtained through legal deductions of interest payments, medical expenses, philanthropic donations, depreciation, amortization, etc. Such deductions reduce part of the taxpayer's taxable income in a given year or transfer tax debts to future years (Fernández, 2004 and 2006). So, the tax benefit reduces the total amount of the company's tax liability; D is the debt value in the current period; NPV is the net present value of stock's capitalization; K_u is the required profitability of the shares of the company not levered (with $D = 0$); K_d is the required profitability of the company's debt (being $K_d = R_F + \beta_d P_M$, where P_M is prime rate market risk and β_d is the beta applied on company's debt); T is the company's tax burden; R_F is the risk-free rate.

$$K_e = K_u + \frac{D(1-T)}{E} (K_u - R_F) \quad (2)$$

Where E is the stock's value and K_e is the required profitability of the company's stocks (or the cost of own resources). It is defined by $K_e = R_F + \beta_L P_M$, being $\beta_L P_M$ the product of levered beta (beta for a company with debt) and prime rate market risk.

The levered beta is given by:

$$\beta_L = \beta_u + \frac{D(1-T)}{E} \beta_u \quad (3)$$

Where β_u is the unlevered beta (beta for a company without debt).

With this, we now define WACC identity:

$$WACC = K_u \left(1 - \frac{DT}{E+D} \right) + \frac{(K_d - R_F)(1-T)}{E+D} \quad (4)$$

And the debt's net present value is deduced as follow:

$$NPV[D_t] = \frac{\Delta D_t - D_{t+1} (K_d - R_F)(1-T) / T}{(1 + K_u)^t} \quad (5)$$

3.1. Data sources, forecasts and cash flow

The model was developed using the Wolfram Mathematica 9.0 software¹ (for the computation of intrinsic value series and for its estimation by a seasonal autoregressive integrated moving average model - SARIMA)², and Excel 2010 (to obtain and store company's accounting reports, as well for plotting charts).

Our model use discounted cash flow techniques for leveraged companies. Its cash flows are constructed from the standardized financial statements in accordance with the rules of CVM.

The following tables were made by using an accounting plan adapted to Brazilian accounting standards and based on the definitions of Fernández (2008), Damodaran (2012) and Copeland, Koller and Murrin (2001).

Table 2 shows the cash flow structure as well the sources of the accounts that make up this flow.

Table 2 - Accounts of operating cash flow

<i>Signal</i>	<i>Cash flow's accounts</i>	<i>Sources</i>
(=)	Operating profit before income tax	Income Statement (IS)
(-)	Taxes on Operating Profit	Income Statement (IS)
(=)	Operating profit after income tax	Authors' calculations
(+)	Depreciation, depletion and amortization – DD&A	Statement of Other Income and Expenses
(-)	Variations in investments in fixed capital	Balance sheet
(-)	Variations in working capital	Balance sheet
(=)	Available operating cash flow	Authors' calculations

¹ About the broad possibilities of different versions of Wolfram Mathematica, see: Varian (1993), Blachman (1992), Stinespring (2002) and Kendrick, Mercado and Amman (2005).

² The seasonality of quarterly data series recommends the use of a SARIMA model. For more details, see: Hamilton (1994, p.167-69 and p. 437-42) and Wolfram's User's Guide of Time Series (<https://reference.wolfram.com/legacy/applications/timeseries/UsersGuideToTimeSeries/NonstationaryAndSeasonalModels/1.3.2.html>).

Source: Elaboration of authors based on Fernández (2008), Damodaran (2012) and Copeland, Koller and Murrin (2001).

Table 3 describes the sources of the model parameters.

Table 3 – Sources of parameters used in the model

<i>Parameters</i>	<i>Meaning and data used</i>	<i>Sources</i>
R_F	R_F = 12-month LTN return - Average IPCA expectations for the next 12 months. LTN is a national treasury bond.	Central Bank of Brazil (Banco Central do Brasil - BCB).
K_d	Required return on company debt.	Authors' calculations.
K_M	K_M = equity risk premium = $R_F + P_M$.	BM&F/Bovespa and BCB
β_i	β_i = quarterly beta of PETR3.	Authors' calculations based on BM&F/Bovespa data
β_d	Beta of the company's debt, given by $K_d = R_F + \beta_d P_M$.	Authors' calculations based on BM&F/Bovespa data
β_u	Beta of unlevered company's stocks, given by $K_u = R_F + \beta_u P_M$.	Authors' calculations based on BM&F/Bovespa data
β_L	Beta of levered company's stocks, given by $K_e = R_F + \beta_L P_M$.	Authors' calculations based on BM&F/Bovespa data
P_M	P_M = Brazilian prime rate (TPB or "taxa preferencial brasileira")	Central Bank of Brazil (Banco Central do Brasil - BCB).
T	Tax burden = total taxes paid / net sales revenue	Petrobras income statement and Exame Magazine's "Best and Bigger Yearbook" ("Melhores e Maiores" da Revista Exame). The tax burden data were obtained from the yearbook and converted into reais at the commercial exchange rate R \$ / US \$ (whose source is IPEADATA).

Source: Elaboration of authors based on Fernández (2008), Damodaran (2012) and Copeland, Koller and Murrin (2001).

Table 4 shows the accounting identities and sources that generated the cash flows.

Table 4 - Forecast of balance sheets (balances) and company profit and loss accounts

<i>Signs</i>	<i>Asset and capital accounts</i>	<i>Formulas</i>	<i>Sources</i>
(-)	NWC (Net Working Capital)	$NWC = \text{Current assets} - \text{Current liabilities}$	Balance Sheet
(-)	Gross fixed assets (GFA)	-	Balance Sheet
(-)	Accumulated amortization, depletion and depreciation (DD&AA)	-	Cash Flow of table 1
(=)	Net Fixed Assets (NFA)	$NFA = TA - NWC - GFA - DD\&AA$	-
(=)	Total Assets (TA)	-	Balance Sheet
	<i>Accounts receivable and shareholders' equity</i>	<i>Formulas</i>	<i>Sources</i>
(=)	Debt	$\text{Debt} = \text{Loans and financing of current and noncurrent liabilities}$	Balance Sheet
(+)	Capital (book value)	$\text{Capital} = \text{TL} - \text{Debt}$	Balance Sheet
(=)	Total liabilities (TL)	-	Balance Sheet
	<i>Profits, expenses and taxes</i>	<i>Formulas</i>	<i>Sources</i>
(=)	Gross profit (GP)	-	Income Statements
(+ or -)	Financial result (FR)	$\text{FR} = \text{Financial income} - \text{Financial expenses}$	Income Statements
(=)	Net income before taxes (NIBT)	$\text{NIBT} = \text{GP} \pm \text{FR}$	
(-)	Tax burden (T)	-	Exame Magazine's "Best and Bigger Yearbook" ("Melhores e Maiores" da Revista Exame). The tax data were converted into reais at the commercial exchange rate R \$ / US \$ (whose source is IPEADATA).
(=)	Net income after taxes (NIAT)	$\text{NIAT} = \text{NIBT} - \text{T}$	-

Source: Elaboration of authors based on Fernández (2008), Damodaran (2012) and Copeland, Koller and Murrin (2001).

Given the data from balance sheets and income statements of table 4 we can obtain the flows of table 5. After this, we have projected this flow for three quarters ahead of the

third quarter of 2016 using a SARIMA model performed in Wolfram Mathematica 9.0¹.

We use the command

$SARIMAProcess[\{a_1, \dots, a_p\}, d, \{b_1, \dots, b_q\}, \{s, \{\alpha_1, \dots, \alpha_m\}, \delta, \{\beta_1, \dots, \beta_r\}\}, v]$ that

represents a seasonal integrated autoregressive moving-average process with ARIMA coefficients a_i , d , and b_j ; seasonal order s ; seasonal ARIMA coefficients α_i , δ , and β_j ; seasonal integration order d ; and normal white noise with variance v^2 .

Finally, we discounted the projected flow results' with the formulas mentioned in the description of the model of Damodaran (1994), as we have seen in section 3.

Table 5 - Projected cash flows

<i>Cash Flow Item</i>	<i>Formulas</i>	<i>Meaning</i>	<i>Source</i>
CFac _t Dividends	$CFac_t = NIBT_t - \Delta NWC_t - \Delta NFA_t + \Delta Debt_t$	Shareholders' expected cash flow in t	Income Statement and table 2
CFd	$CFd = (D \cdot Kd) - \Delta D$	Debt's expected cash flow.	Table 2. Parameter Kd was computed by authors as mentioned in Table 3.
FCF	$FCF = CFAct - \Delta D - [D \cdot Kd(1 - T)]$	Free cash flow	Table 2. Parameter Kd was computed by authors as mentioned in Table 3. See Tables 3 and 4 for the sources of T (the company's tax burden).

Source: Elaboration of authors based on Fernández (2008), Damodaran (2012) and Copeland, Koller and Murrin (2001).

Note: In case of unlevered company we have $FCF = CFac$.

4. Results

In the first subsection, we analyze the factors that determined the profile of the final quarterly series of intrinsic values (IVs) and closing prices of PETR3. This series was obtained by the application of Damodaran's model.

¹ About the broad possibilities of different versions of Wolfram Mathematica, see: Varian (1993), Blachman (1992), Stinespring (2002) and Kendrick, Mercado and Amman (2005).

² For the sake of clarity and concision, the SARIMA model results are available at: <http://marcelopa.dominiotemporario.com/Pesquisa.php>

In the second, we verify the annual changes in value in relation to the previous year of Petrobras in 2011-2016. We also measure the state company's loss of value along these years.

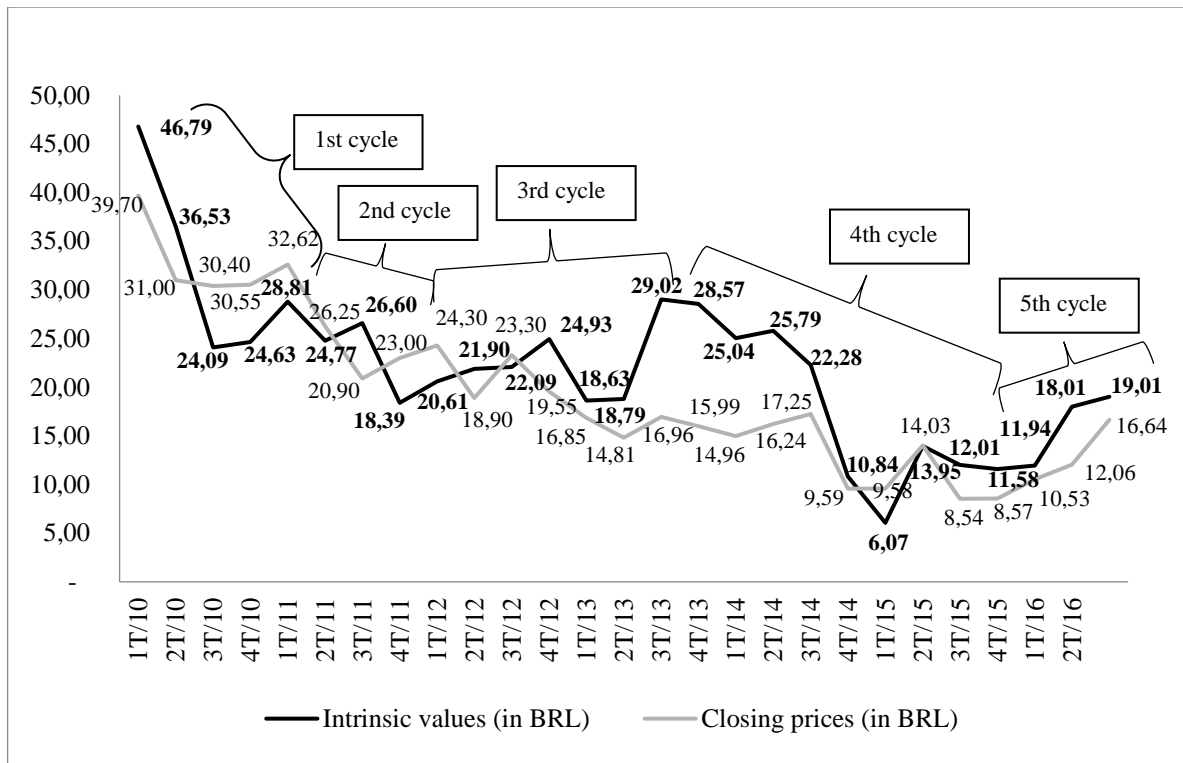
4.1. Examining Petrobras' intrinsic values' series

In Figure 1, it is possible to observe five cycles of PETR3 intrinsic values' (IVs) and closing prices' behavior.

In the first cycle (Q1-10 / Q4-10) there was a downside period (prices > IVs) within a downtrend of both curves. Brent-type barrel prices rose 15.81%, which may have influenced the optimism of the corporation's planning expressed by then-president José Gabrielli:

“Petrobras is pressing ahead with an ambitious plan to invest \$174 billion over the next five years. Critics question how much of this will materialise. But Mr Gabrielli says that even if the oil price falls to no more than \$45 a barrel, the \$30 billion that he has raised this year will be enough to fund plans for the next two years. If the oil price is at \$65 a barrel (where it stood this week) the company can fund plans for the next five years. In addition to developing the new fields, it wants to build five new refineries, to generate more electricity and to build a network of gas pipelines in Brazil.” (The Economist, 07/23/2009).

Figure 1 - Intrinsic values and closing prices of PETR3 (in BRL) according to Damodaran's model (1994): 1st quarter/2010 to 3rd quarter/2016



Source: Computed by the authors.

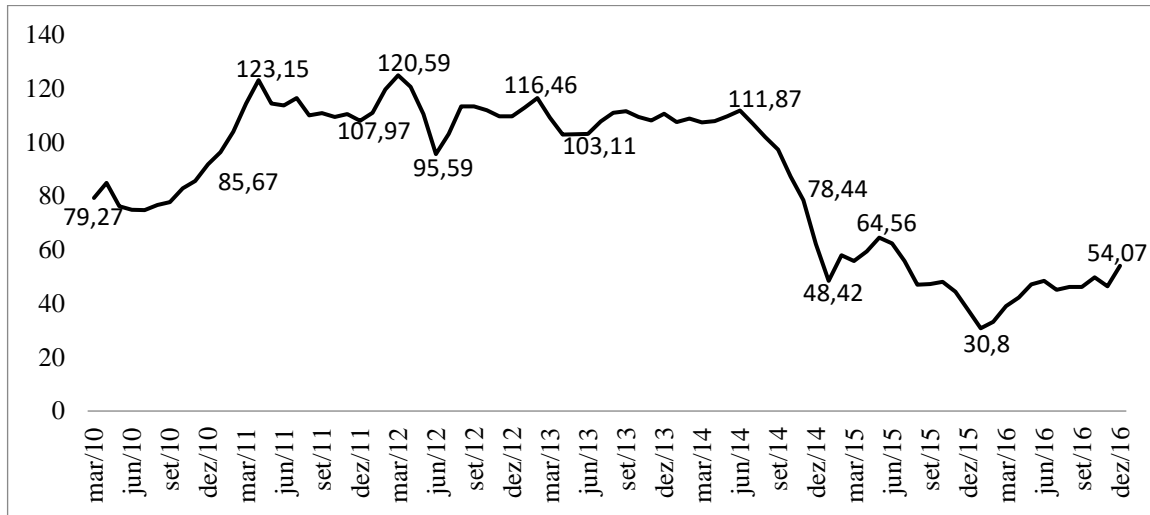
It is known that, unlike José Gabrielli's optimistic forecasts, Brent barrel prices declined 31.79% in Q1-10 to Q3-16 (figure 2). The investment plan has not only become unfeasible, as well has generated a large indebtedness. This negatively affects the company at least until the date of finalization of this work. The current president Pedro Parente tries to deal with this problem leading a more technical management and aimed at administrative efficiency and cost reduction.

In the second cycle (Q1-11 to Q4-11) there were oscillations that revealed downsides (closing prices > IV's) and upsides (reverse situation). But these oscillations still continued in a downward trend of both curves. Brent barrel prices rose 12.13% in this cycle (figure 2) and were not the cause of this downward bias. The facts that contributed to such behavior of PETR3 and its IVs refer to a Petrobras' error of estimation about global oil prices and the international supply and demand conditions of this commodity (The Economist, 02/09/2010).

In the third cycle (Q1-12 through Q3-13) the downside situation and declining trajectory of the two curves was still predominant. Brent oil prices oscillated a bit more. But they did not decline and varied by 0.568% from the beginning to the end of this cycle. Meanwhile, there was an important factor that began to negatively affect the finances of the company: the discretion of the Executive in the control of administered prices,

especially those of petroleum products. The non-declared goal was to control the persistent growth of inflation rates. Besides that, the situation of Petrobras was aggravated by the first denunciations of Operation Carwash (Operação Lava Jato).

Figure 2 - Barrel of Brent crude oil (US\$ / barrel)



Source: World Bank.

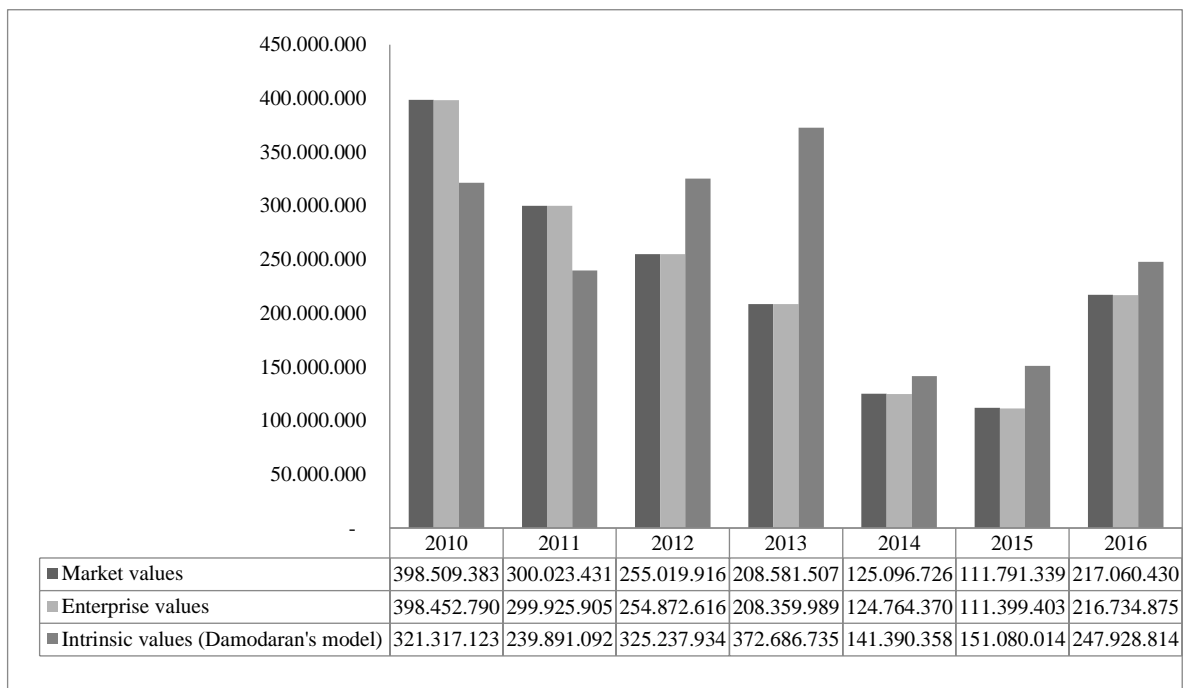
In the fourth cycle (Q4-13 to Q4-15), there was clearly alternations of upsides and downsides, reflecting the uncertainty of the period, also marked by the losses of the IVs and declines in the share closing prices. During this period, Petrobras recorded historical losses, driven by the continuity of the fuel price containment policy, studied by Azevedo and Serigatti (2015), by the drop in Brent prices on the international market (figure 2) and by the continuous complaints of corruption involving company directors, key politicians and contractors that were investigated and arrested by Operation Carwash.

In the fifth cycle (Q1-16 to Q3-16), PETR3 finally shows a resumption of its closing prices and IVs. The stock revealed more continuous downsides, motivated by the recovery of company's productive activity, costs' minimizations and measures to reduce its indebtedness. The company, when we conclude this article, gradually undergoes a process of restoring investor confidence after Pedro Parente assumed the presidency. On October 21, 2016, Moody's improved Petrobras' rating from "B3" to "B2", changing its outlook from negative to stable. However, the oil company still remains far from investment grade rating.

4.2. Measuring the company's loss of value

Figure 4 details the changes in relation to the previous year in three Petrobras value indicators from 2011 to 2016. The intrinsic value of the company was obtained by multiplying the intrinsic values of PETR3 and PETR4 and their respective weights by the total number of shares of the oil company. The market value correspond to the product of the closing prices by this total number. The enterprise value (EV) is defined by financial accounting as the amount necessary to acquire a company and pay off its debts and is given by the following accounting identities (which are tautological and therefore reach the same value): a) $EV = \text{Capitalization or Market Value} + \text{Debt (Long and Short Term)} - \text{Cash and Equivalents}$; B) $EV = \text{Capitalization or Market Value} - \text{Net Debt}$; And c) $\text{Net Debt} = \text{Loans and Financing of Current Liabilities} + \text{Loans and Financing of Non-Current Liabilities} - \text{Cash and Cash Equivalents} - \text{Financial Investments of Current Assets} - \text{Financial Investments Valued at Fair Value (Non-Current Assets)}$.

Figure 4 - Changes in value (BRL) in relation to the previous year of Petrobras in 2011- 2016



Sources: Authors' calculations based on the model of agents and 81 financial statements of Petrobras

The oil company lost BRL 46.43 billion in market value in 2013 compared to the previous year. And it still lost about BRL 231.29 billion in intrinsic value in 2014 in relation to 2013. After having registered substantial losses by the three value indicators

in 2015, the company shows a recovery in its intrinsic values (BRL 96.84 billion), market (BRL 105.26 billion) and enterprise value (R\$ 105.34 billion) in the three quarters of 2016 analyzed.

Therefore, the amount of oil state company's losses of value was, for the period of 2010 to the third quarter of 2016: BRL 73.38 billion in intrinsic value; BRL 171.71 billion in enterprise value and BRL 181.44 billion in market value.

5. Final remarks

Value destruction of Petrobras, the largest Brazilian company, in market value in 2013 compared to the previous year, reached R \$ 46.43 billion. In addition, its intrinsic value calculated by Damodaran model (2014) was reduced by approximately R \$ 231.29 billion in 2014 compared to 2013. In 2015, after significant losses of value, measured by the three criteria, the state oil company showed some recovery of its intrinsic value (R \$ 96.84 billion), as well market value (R \$ 105.26 billion). The same occurred with the company's value (R \$ 105.34 billion) in the three quarters of 2016.

The company value's losses was, for the period from 2010 to the third quarter of 2016: BRL 73.38 billion in intrinsic value; BRL 171.71 billion in enterprise value and BRL 181.44 billion in market value.

It should be noted, despite the severe measured losses, the 2016 "turnaround" in the company's value generation trajectories was, besides the improvement of the expectations after Pedro Parente becomes the CEO, due to the fact that the company's debt reduction policy was positively perceived by investors and Petrobras stakeholders.

Another important factor was the approval of the bill in the Federal Senate that allows Petrobras to choose the pre-salt exploration projects it intends to participate in. The bill, which is authored by José Serra, a senator and former foreign minister, is under discussion in the Brazilian Congress and modifies the current law that requires the company to hold a minimum of 30% of all consortiums that exploit these reserves. The current law also guarantees her the right to exclusivity as a pre-salt operator. If the bill is approved, other oil companies, including foreign ones, will be able to exploit the pre-salt, but only in fields where Petrobras has no interest and/or investment capacity to explore.

Approval of this bill in the Senate helped to reduce the mistrust that investors in PETR3 have shown over most of Dilma Rouseff's presidential administration.

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